CS171 HW 5 - NURBS Editor

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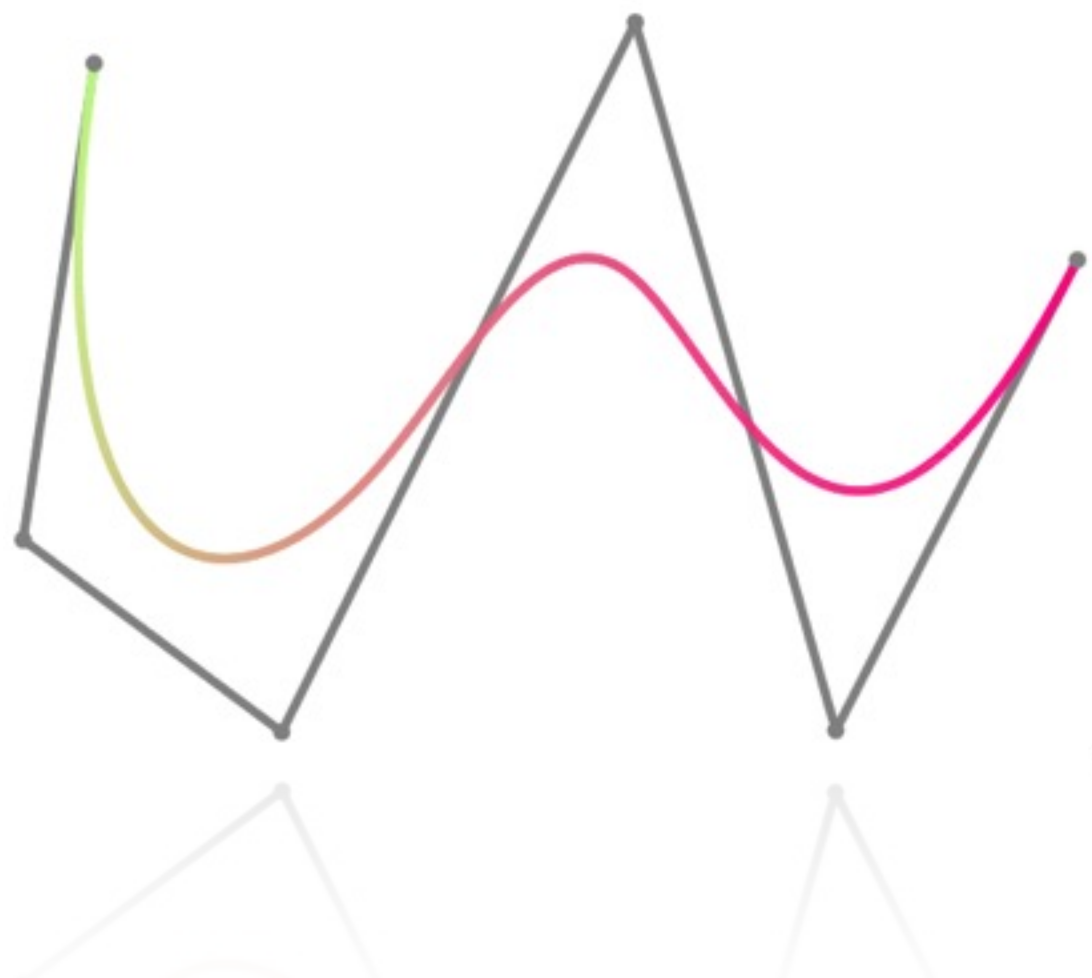
Based on Slides by Tom Raney (11/9/07)

NURBS

- Non-Uniform Rational B-Splines
- Representation of smooth curves (C^2!)
- Intuitive, flexible, precise
- Generalization of Bezier paths (uniform, non-rational b-splines)

General Idea

- Control points, p_i, $i \in \{0, ..., n\}$
- Parameterized by $u \in [0, 1]$
- Curve $Q(u)$: $u \to (x, y)$
    - Blending of control points



How are they blended?

- Cox de Boor's Algorithm

$$Q(u) = \sum_{i=0}^{n} p_i N_{i,k}(u)$$

- Control points, p_i, $i \in \{0, ..., n\}$
  - (x, y) coordinates controlling curve shape
- Parameter $u \in [0, 1]$
- k = curve order = degree + 1
  - For cubics, k = 3 + 1 = 4

Knots

- Nondecreasing sequence t_i $i \in \{0, ..., n+k\}$
- All t_i are in the parameter space ([0, 1])
- Intuitively: specify what portions of the curve the control points affect. For instance:

$$u \notin [t_i, t_{i+k}] \Rightarrow N_{i,k} = 0$$

- (Control p_i only affects curve segment over $[t_i, t_{i+k}]$)

Knots

- Invisible to user of your program!
- Integral part of $N_{i,k}$ computation
- Note, there are n + k + 1 knots, or:
  - # Knots = # Control Points + Order

What's this magic $N_{i,k}$?

- Recursively defined:

$$N_{i,1}(u) = \begin{cases} 1 & t_i \leq u \leq t_{i+1} \\ 0 & otherwise \end{cases}$$

$$N_{i,k}(u) = \frac{(u - t_i)N_{i,k-1}(u)}{t_{i+k-1} - t_i} + \frac{(t_{i+k} - u)N_{i+1,k-1}(u)}{t_{i+k} - t_{i+1}}$$

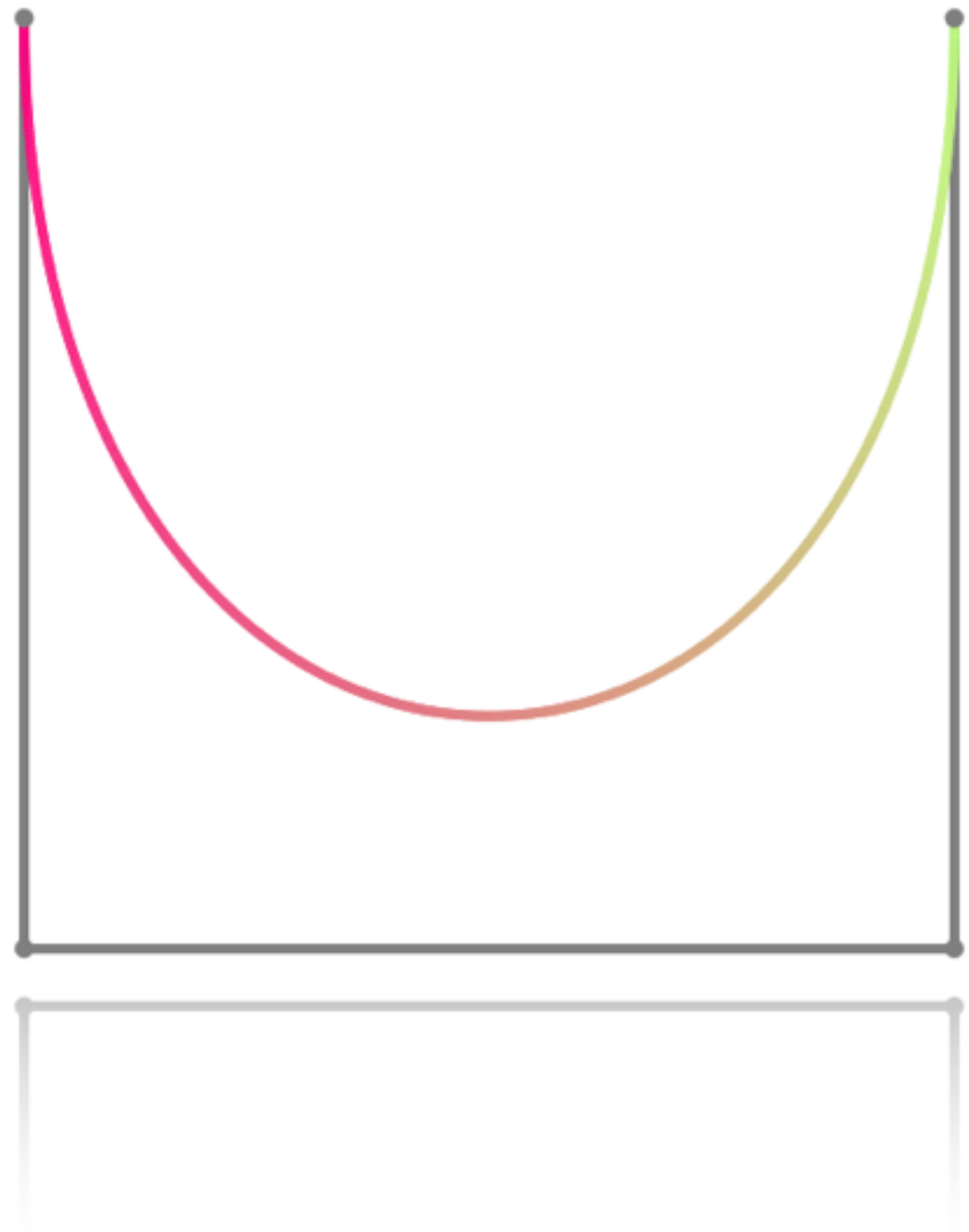
- To handle repeated knots ($t_i = t_{i+1}$) we must define 0/0 = 0 or we get an undefined result
  - You will need to do this!

Knot Multiplicity

- To get the curve to end at the last control point, you must have endpoints of multiplicity k
- For cubic splines (k = 4), this means:
    - knots = [0, 0, 0, 0, t_k, ..., t_n, 1, 1, 1, 1]

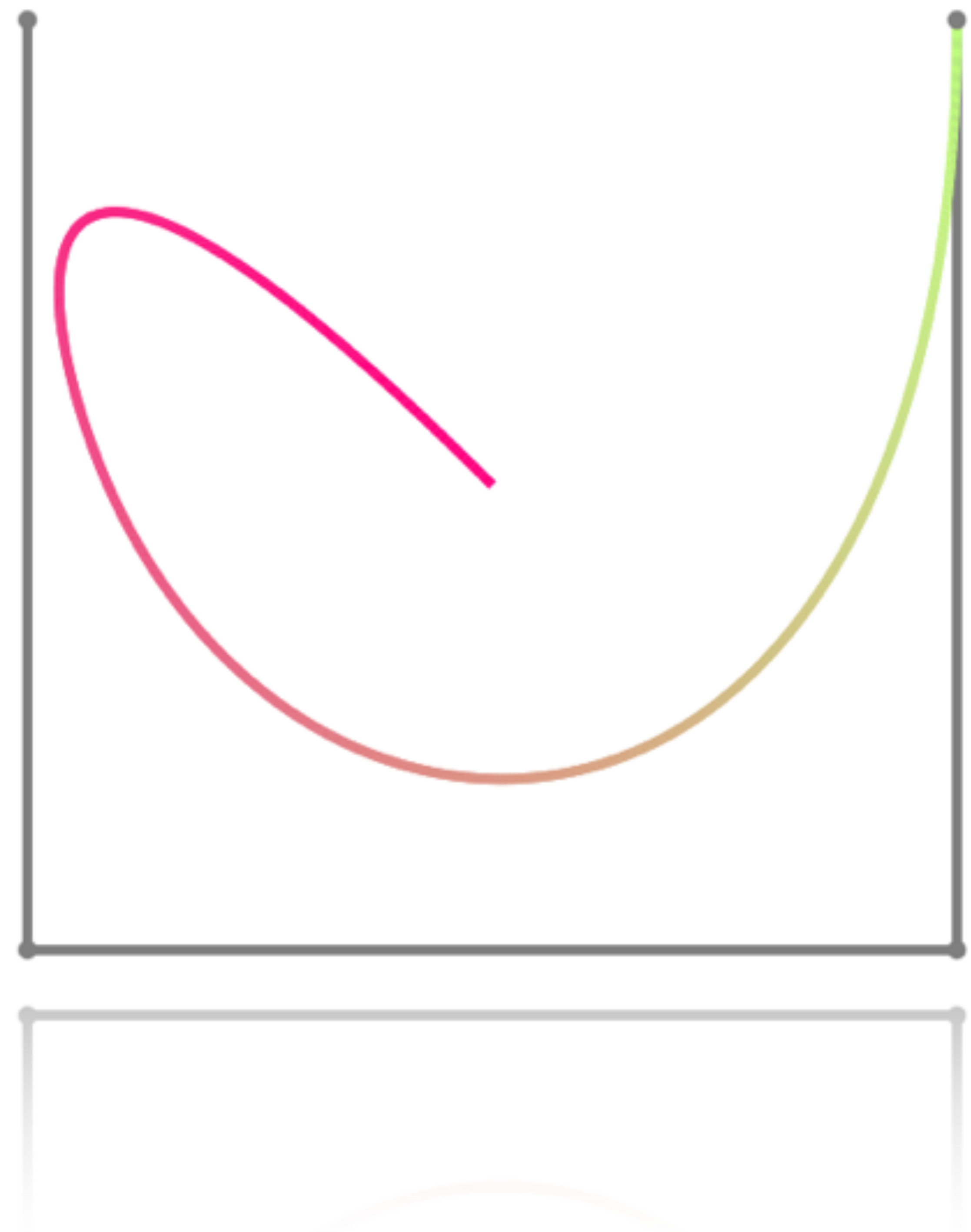
Simple Example

- 4 control points
- n + k + 1 = 8 knots
- knots = [0, 0, 0, 0, 1, 1, 1, 1] for curve to meet endpoints
- This is your initial editor configuration!



I'm not lying about end knot multiplicity

- What happens when we don't make the end knots of multiplicity k?
- Take [0, 0, 0, .25, 1, 1, 1, 1] as the knot vector
- Uh-oh!



Drawing Curves

- Keep track of control points and knots
- Choose a step size, Δu, small enough to get a smooth curve
  - Adaptive Δu is extra credit!
  - Draw short line segments from $Q(u)$ to $Q(u + \Delta u)$

Inserting Control Points

- We don't want to alter the curve!
- Old control points will have to move...
- Another control point means another knot. Insert a knot with the u-value of the mouse click in the **correct position** of the knot array!
- Then create the new control point and compute positions:

$$p'_{n+1} = p_n \qquad p'_0 = p_0$$

$$p'_i = (1 - a_i)p_{i-1} + a_i p_i$$

$$a_i = \begin{cases} 1 & i = 1, ..., j-k \\ \frac{t' - t_i}{t_{i+k} - t_i} & i = j-k+1, ..., j \\ 0 & i = j+1, ..., n \end{cases}$$

Note: Assignment page had the order of this procedure incorrect until this morning.

Questions?

- Implementation is straightforward.
- I've tried to fix all of the old, confusing errors on the assignment page. Please let me know if you find any more, though!
- This was a very practical overview--read through some of the linked references for a deeper discussion of the math!